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FUZZY COLOUR FILTERING USING RELATIVE ENTROPY

A. Fotinos, N. Laskaris and S. Fotopoulos
University of Patras, Electronics Laboratory, 26110 Patras, Greece
e-mail: spiros@physics.upatras.gr

ABSTRACT

Relative entropy (**E_{rel}**) has long been used as a distance measure between two sets of sampled data. In this work **E_{rel}** is used for filtering of colour images. Local statistical characteristics of the image are estimated by means of Parzen estimators. Two indices for the noise are derived. These indices are associated with membership functions. Filtering of the image is achieved by using a small number of fuzzy rules.

INTRODUCTION

Fuzzy logic is a very well suited approach to image processing, taking into account at the same time several factors and imaging conditions. One of the main factor affecting the filtering process is the type of noise existed in the image. Most of the efforts being done in recent years aim in finding at each case the most appropriate filter type. Adaptive techniques have also been produced. Another factor always taken into consideration is the filter response to local image characteristics which have to be kept intact. Fuzzy logic techniques provide an effective tool to handle all these situations straightforward and in a rather simple way. Application of this technique in gray scale images has already been presented [3]. In this paper our efforts are focused on color images where the strong correlation of the R,G,B channels has to be considered. Multivariate processing techniques are dominating in this field and are also employed here.

In this work two parameters are defined in order to control the filtering process. The first one is the local Relative Entropy (E_{rel}) between a known probability distribution f(**X**) and the local probability distribution p(**X**). Relative Entropy originally was introduced by Kullback [6] and is defined as following, (for discrete data):

$$E_{rel}(p,f)=\sum_{X} p(\mathbf{X}) \cdot \log\left(p(\mathbf{X})/f(\mathbf{X})\right) \quad (1)$$

Other definitions could also be found in the literature [1,2]. The above definition easily explains why E_{rel} is used as a measure of similarity or better as a distance between the two distributions.

In order to apply the idea of E_{rel} to color images, an estimation of the probability density p(**X**) should precede. In this work the estimation of p(**X**) in a position **X** of RGB space is performed by means of potential functions (PF) or Parzen estimators [5] as follows:

$$p(\mathbf{X})=\frac{1}{N h^{p}} \sum_{i=1}^{N} K\left(\frac{\mathbf{X}-\mathbf{X}_{i}}{h}\right) \quad (2)$$

where K(·) is a Gaussian kernel, N the window size and $\mathbf{X}_i$ the RGB vectors inside the local window. Having estimated p(**X**) based on the vectors X_i we compute $E_{rel}(p,f)$**.** In the summation, special care is taken to include not only values of p(**X**) for the window points $\mathbf{X}_i$ but for more points of the RGB space, in order to sample the space as uniformly as possible. So we must find a number of new vectors within the range of the real vector samples of the window. This process is done at each position of processing window and a new set of samples is used to find the probability distributions, p(**X**) and f(**X**). If the f(**X**) is a Gaussian distribution, with known values of mean and variance, $E_{rel}(p,f)$ will take small values when the samples of the window belong to a homogeneous region.

However the presence of an impulse added on the Gaussian noise does not increase too much the E_{rel} and could lead to wrong conclusions about the nature of the noise and consequently for the filter to be employed. Therefore a second index is introduced for impulse detection of the central vector sample of the window. Among the various methods available in the recent literature we use the following :

$$I_d = \left| \overline{\mathbf{X}} - \left(\frac{\sum_{i=1}^{N} \mathbf{X}_i - \mathbf{X}_c}{\mathbf{N-1}} \right) \right| \tag{3}$$

where $\overline{\mathbf{X}}$, X_c are the mean value of the window vectors $\mathbf{X_i}$ and the center of window vector respectively. Small values of this index guarantee the absence of impulse. We have to notice that impulse detection could also be done with an appropriate form based on relative entropy.

THE FILTERING PROCESS

The information provided by Relative Entropy E_{rel} is used to make assumption about the nature of each part of the image. Specifically E_{rel} as a distance measure between the local pdf and a Gaussian (provided as a global information of the image), is an index of how close this part is to a constant region with Gaussian type noise distribution.

Continuing the classification of the image regions large values of E_{rel} are associated with edges or ramps. In this paper, we do not discriminate between them but an extension of the method to that direction is straightforward.

Using the above two indices the image regions are perceptually classified as follows:

- The first class is a homogenous region, where E_{rel} as well as I_d take small values
- The second class is a edge region, where E_{rel} takes large values and I_d small
- The third class is a homogeneous region with an impulse in the central sample of the window. In this case E_{rel} takes small values and I_d large.
- The forth class is a edge region and the central sample of the window is impulse. Here E_{rel} as well as I_d take large values

Image enhancement is accomplished by adopting the vectorial a-trimmed filter, which is given as following [7]:

$$\mathbf{Y}_\alpha = \frac{1}{\mathbf{N}(1-2\alpha)} \sum_{\mathbf{i}=1}^{\mathbf{N}(1-2\alpha)} \mathbf{X_i} \tag{4}$$

where X_i are the i-th vector ordered sample and α is a constant. Vector ordering techniques are assumed. For better results and in the case of an edge region the current sample could also be selected if it is not an impulse.

Table I

Class	Appropriate Filter
1	$\mathbf{y_1 = Y_{0.055}}$
2	$\mathbf{y_2} = (\mathbf{Y_{0.400}} + \mathbf{X_{center}})/2$
3	$\mathbf{y_3 = Y_{0.200}}$
4	$\mathbf{y_4 = Y_{0.333}}$

The appropriate filter for each class is given in table I. This filter selection is not unique, other techniques for each class with lower computational efficiency could also be used.

DEFINITION OF FUZZY SETS AND FUZZY RULES

In this paper two fuzzy sets, *Small* and *Large*, are proposed for the relative entropy E_{rel} as well as for the impulse detector I_d. They are shown in figure 1. The *Large* fuzzy set is the complementary of *Small*.

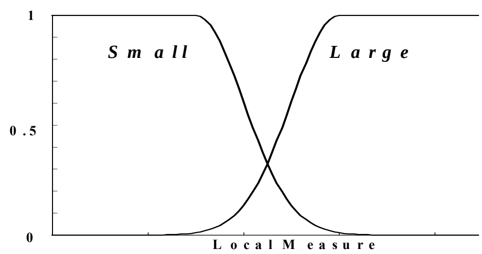

Fig. 1. Two fuzzy sets for both E_{rel} and I_d are considered Small and Large.

We assume $\mu_S(E_{rel})$, $\mu_L(E_{rel})$, $\mu_S(I_d)$, $\mu_L(I_d)$ membership values for the *Small* and *Large* sets of E_{rel} and I_d.

Now we can define the four rules (one for each class) by the above two indices, which are presented in Table II.

Table II

IF	E_{rel} is *Small* **AND** I_d is *Small*	**THEN**	Class 1
IF	E_{rel} is *Large* **AND** I_d is *Small*	**THEN**	Class 2
IF	E_{rel} is *Small* **AND** I_d is *Large*	**THEN**	Class 3
IF	E_{rel} is *Large* **AND** I_d is *Large*	**THEN**	Class 4

The inference result for each rule is given by translating the AND as the minimum operation.
Finally the overall output is constructed by the following defuzzification operation which is based on the weighted average method:

$$\mathbf{Y} = \frac{\sum_{\mathbf{k}=1}^{5} \mu_{\mathbf{k}}(\mathbf{X}) \cdot \mathbf{y}_{\mathbf{k}}(\mathbf{X})}{\sum_{\mathbf{k}=1}^{5} \mu_{\mathbf{k}}(\mathbf{X})} \qquad (5)$$

where μ_k is the total weight of k-th rule and $y_k(\mathbf{X})$ the output of the suitable filter (see table I).

EXPERIMENTAL RESULTS

The proposed algorithm has been implemented for the PEPPERS image (true color 24-bit per pixel). The noise added to the original image is mixed gaussian$(0,15^2)$ and impulsive(1%) for each channel. The proposed algorithm is also compared to the VMF (Vector Median Filter) and the average filter of the same length. Comparisons are carried out by means of the Mean Squared Error computed as follows:

$$\mathbf{MSE} = \frac{1}{3MN} \sum_{\mathbf{i}=1}^{N} \sum_{\mathbf{j}=1}^{M} \left\| \mathbf{Y}(\mathbf{i,j}) - \mathbf{X}(\mathbf{i,j}) \right\|^2$$

where **Y(i,j)** are the pixels of the output image, **X(i,j)** are the pixels of ideal image and N, M are the image dimensions (in pixels).

Table III

Filter	**σ=15 & 1%**	**σ=20 & 2%**
Vector Median	234	247
Averager	143	195
Proposed	97	143

In figure 2 the output images of the above comparison are also given. These figures as well as the numbers of TABLE III indicate that the proposed filters are working very well and are comparable to the existing classical logic algorithms .

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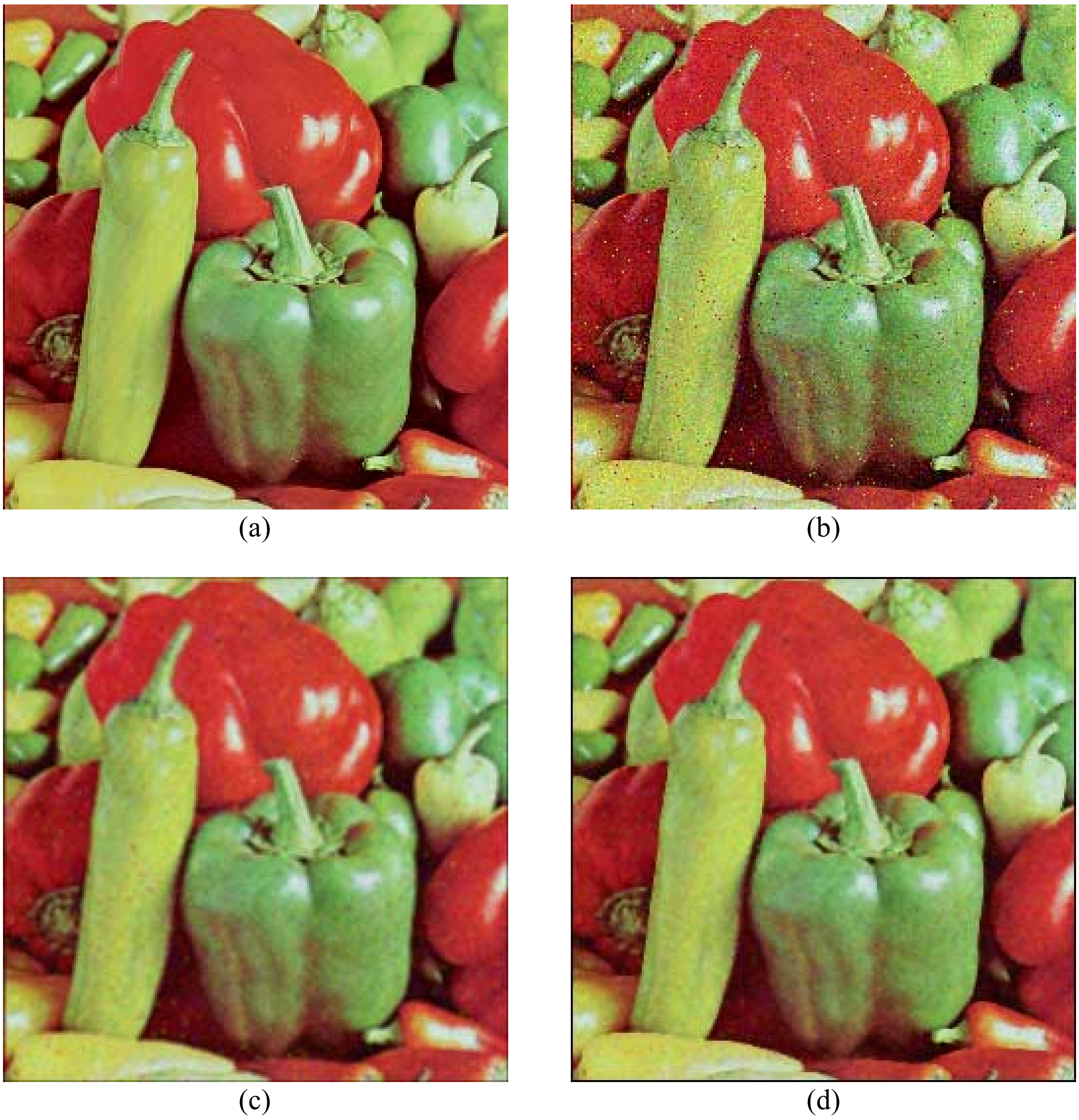

Fig.2. (a) The "Peppers", 256x256, 24-bit per pixel, (b)Noisy Image, (c) the output of the Averager and (d) the output of the proposed filter.

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