FUZZY COLOUR FILTERING USING RELATIVE ENTROPY

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ABSTRACT
Relative entropy ($E_{rel}$) has long been used as a distance measure between two sets of sampled data. In this work $E_{rel}$ is used for filtering of colour images. Local statistical characteristics of the image are estimated by means of Parzen estimators. Two indices for the noise are derived. These indices are associated with membership functions. Filtering of the image is achieved by using a small number of fuzzy rules.

INTRODUCTION
Fuzzy logic is a very well suited approach to image processing, taking into account at the same time several factors and imaging conditions. One of the main factor affecting the filtering process is the type of noise existed in the image. Most of the efforts being done in recent years aim in finding at each case the most appropriate filter type. Adaptive techniques have also been produced. Another factor always taken into consideration is the filter response to local image characteristics which have to be kept intact. Fuzzy logic techniques provide an effective tool to handle all these situations straightforward and in a rather simple way. Application of this technique in grayscale images has already been presented [3]. In this paper our efforts are focused on color images where the strong correlation of the R,G,B channels has to be considered. Multivariate processing techniques are dominating in this field and are also employed here.

In this work two parameters are defined in order to control the filtering process. The first one is the local Relative Entropy ($E_{rel}$) between a known probability distribution $f(X)$ and the local probability distribution $p(X)$. Relative Entropy originally was introduced by Kullback [6] and is defined as following, (for discrete data):

$$E_{rel}(p,f)=\sum_{X} p(X) \cdot \log(p(X)/f(X))$$  \hspace{1cm} (1)

Other definitions could also be found in the literature [1,2]. The above definition easily explains why $E_{rel}$ is used as a measure of similarity or better as a distance between the two distributions.

In order to apply the idea of $E_{rel}$ to color images, an estimation of the probability density $p(X)$ should precede. In this work the estimation of $p(X)$ in a position $X$ of RGB space is performed by means of potential functions (PF) or Parzen estimators [5] as follows:

$$p(X)=\frac{1}{Nh^p} \sum_{i=1}^{N} K\left(\frac{X-X_i}{h}\right)$$  \hspace{1cm} (2)

where $K(\cdot)$ is a Gaussian kernel, $N$ the window size and $X_i$ the RGB vectors inside the local window. Having estimated $p(X)$ based on the vectors $X_i$, we compute $E_{rel}(p,f)$. In the summation, special care is taken to include not only values of $p(X)$ for the window points $X_i$ but for more points of the RGB space, in order to sample the space as uniformly as possible. So we must find a number of new vectors within the range of the real vector samples of the window. This process is done at each position of processing window and a new set of samples is used to find the probability distributions, $p(X)$ and $f(X)$. If the $f(X)$ is a Gaussian distribution, with known values of mean and variance, $E_{rel}(p,f)$ will take small values when the samples of the window belong to a homogeneous region.

However the presence of an impulse added on the Gaussian noise does not increase too much the $E_{rel}$ and could lead to wrong conclusions about the nature of the noise and consequently for the filter to be employed. Therefore a second index is introduced for
impulse detection of the central vector sample of the window. Among the various methods available in the recent literature we use the following:

$$I_d = \frac{\sum_{i=1}^{N} X_i - X_c}{N-1}$$  \hspace{1cm} (3)$$

where $\bar{X}$, $X_c$ are the mean value of the window vectors $X_i$ and the center of window vector respectively. Small values of this index guarantee the absence of impulse. We have to notice that impulse detection could also be done with an appropriate form based on relative entropy.

**THE FILTERING PROCESS**

The information provided by Relative Entropy $E_{rel}$ is used to make assumption about the nature of each part of the image. Specifically $E_{rel}$ as a distance measure between the local pdf and a Gaussian (provided as a global information of the image), is an index of how close this part is to a constant region with Gaussian type noise distribution.

Continuing the classification of the image regions large values of $E_{rel}$ are associated with edges or ramps. In this paper, we do not discriminate between them but an extension of the method to that direction is straightforward.

Using the above two indices the image regions are perceptually classified as follows:

- The first class is a homogenous region, where $E_{rel}$ as well as $I_d$ take small values
- The second class is an edge region, where $E_{rel}$ takes large values and $I_d$ small
- The third class is a homogeneous region with an impulse in the central sample of the window. In this case $E_{rel}$ takes small values and $I_d$ large.
- The fourth class is an edge region and the central sample of the window is impulse. Here $E_{rel}$ as well as $I_d$ take large values

Image enhancement is accomplished by adopting the vectorial a-trimmed filter, which is given as following [7]:

$$Y_{\alpha} = \frac{1}{N(1-2\alpha)} \sum_{i=1}^{N(1-2\alpha)} X_i$$  \hspace{1cm} (4)$$

where $X_i$ are the i-th vector ordered sample and $\alpha$ is a constant. Vector ordering techniques are assumed. For better results and in the case of an edge region the current sample could also be selected if it is not an impulse.

**Table I**

<table>
<thead>
<tr>
<th>Class</th>
<th>Appropriate Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$y_1 = Y_{0.055}$</td>
</tr>
<tr>
<td>2</td>
<td>$y_2 = (Y_{0.400} + X_{center})/2$</td>
</tr>
<tr>
<td>3</td>
<td>$y_3 = Y_{0.200}$</td>
</tr>
<tr>
<td>4</td>
<td>$y_4 = Y_{0.333}$</td>
</tr>
</tbody>
</table>

The appropriate filter for each class is given in table I. This filter selection is not unique, other techniques for each class with lower computational efficiency could also be used.

**DEFINITION OF FUZZY SETS AND FUZZY RULES**

In this paper two fuzzy sets, *Small* and *Large*, are proposed for the relative entropy $E_{rel}$ as well as for the impulse detector $I_d$. They are shown in figure 1. The *Large* fuzzy set is the complementary of *Small*.

![Fuzzy Sets](image)

Fig. 1. Two fuzzy sets for both $E_{rel}$ and $I_d$ are considered Small and Large.

We assume $\mu_S(E_{rel}), \mu_1(E_{rel}), \mu_S(I_d), \mu_1(I_d)$ membership values for the *Small* and *Large* sets of $E_{rel}$ and $I_d$.

Now we can define the four rules (one for each class) by the above two indices, which are presented in Table II.
The inference result for each rule is given by translating the AND as the minimum operation. Finally the overall output is constructed by the following defuzzication operation which is based on the weighted average method:

\[ Y = \frac{1}{\sum_{k=1}^{5} \mu_k(X)} \sum_{k=1}^{5} \mu_k(X) \cdot y_k(X) \] (5)

where \( \mu_k \) is the total weight of k-th rule and \( y_k(X) \) the output of the suitable filter (see table I).

**EXPERIMENTAL RESULTS**

The proposed algorithm has been implemented for the PEPPERS image (true color 24-bit per pixel). The noise added to the original image is mixed gaussian(0,15^2) and impulsive(1%) for each channel. The proposed algorithm is also compared to the VMF (Vector Median Filter) and the average filter of the same length. Comparisons are carried out by means of the Mean Squared Error computed as follows:

\[ \text{MSE} = \frac{1}{3MN} \sum_{i=1}^{N} \sum_{j=1}^{M} [Y(i,j) - X(i,j)]^2 \]

where \( Y(i,j) \) are the pixels of the output image, \( X(i,j) \) are the pixels of ideal image and \( N, M \) are the image dimensions (in pixels).

### Table III

<table>
<thead>
<tr>
<th>Filter</th>
<th>( \sigma=15 ) &amp; 1%</th>
<th>( \sigma=20 ) &amp; 2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector Median</td>
<td>234</td>
<td>247</td>
</tr>
<tr>
<td>Averager</td>
<td>143</td>
<td>195</td>
</tr>
<tr>
<td>Proposed</td>
<td>97</td>
<td>143</td>
</tr>
</tbody>
</table>

In figure 2 the output images of the above comparison are also given. These figures as well as the numbers of TABLE III indicate that the proposed filters are working very well and are comparable to the existing classical logic algorithms.

**REFERENCE**


Fig.2. (a) The “Peppers”, 256x256, 24-bit per pixel, (b) Noisy Image, (c) the output of the Averager and (d) the output of the proposed filter.

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