Fuzzy Color Filtering using potential functions

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Abstract: In this paper a color filtering technique is proposed using fuzzy logic ideas and based on local dispersion and potential function. Two fuzzy sets for each fuzzy variable are considered. These fuzzy variables control the partition of the image to appropriate classes. The parameters of the fuzzy sets are tuned experimentally. Application in color images ('pepper') show that these nonlinear filters suppress gaussian as well as impulsive noise very well.

Introduction

Fuzzy logic techniques in processing of gray-scale images have recently been presented in the literature [1]-[4]. Fuzzy logic is a very well suited approach to color image processing too, taking into account at the same time several factors and imaging conditions. One of the main factor affecting the filtering process is the type of noise overlaid on the image. Most of the efforts being done in recent years aim in finding at each case the most appropriate filter type. Adaptive techniques have also been produced. Another factor always taken into consideration is the filter response to local image characteristics which have to be kept intact. Fuzzy logic techniques provide an effective tool to handle all these situations straightforward and in a rather simple way. In this paper our efforts are focused on color images where the strong correlation of the R,G,B channels has to be considered. Vector processing techniques are dominating in this field and are also employed here.

In this work two parameters are derived in order to control the filtering process. The first one is the local dispersion itself. The second is related to potential functions. Local dispersion v**(X)** provides information for the busyness of the region which could be attributed either to noise or edge. The local dispersion is estimated within an N points window:

$$\mathrm{v(X)} = \frac{1}{\mathrm{N}}\sum_{n=1}^{\mathrm{N}}(\mathrm{X}_i - \overline{\mathrm{X}})^2 \qquad (1)$$

The potential function P**(X)** describes the local concentration and have also been used for the probability density estimation (Parzen estimators) [8]. The Potential P(**X**) produced by the N vectors $\mathbf{X_i}$ in position **X,** is computed as follows:

$$\mathrm{P}(\mathbf{X}) = \frac{1}{\mathrm{N}h^{\mathrm{p}}}\sum_{i=1}^{\mathrm{N}}\mathrm{K}\left(\frac{\mathbf{X} - \mathbf{X}_i}{h}\right) \qquad (2)$$

where p is the vectors' dimension (in our case it's 3) and *h* is a suitably chosen positive parameter. A common choice for the radial function K(**X**) is the multivariate Gaussian function:

$$\mathrm{K}(\mathbf{X}) = (2\pi)^{-p/2}\exp(-\mathbf{X}^2/2) \qquad (3)$$

h is a parameter defining the radius of influence and its optimum value for the case of 3-D Gaussian *f*(**X**) with zero mean and dispersion σ^2, takes the value [7]:

$$h = \sigma\left(\pi^2 2^{3/2} 3\mathrm{N}\right)^{-1/7} \qquad (4)$$

Based on this function the parameter used here is defined as follows [5]:

$$\mathrm{g(X)} = \frac{1}{\mathrm{N}}\sum_{i=1}^{\mathrm{N}}\mathrm{P}(\mathrm{X}_i) \qquad (5)$$

Here g(X) is treated just as a Potential Index is not given any probability meaning to P(**X**) and therefore the normalization constants in (2) are omitted. The maximum value of g(X) in (5) is N. This happens in noiseless flat regions of the image.

This parameter g(X) provides information not only for the spread but also for the structure of the data [5]. It should be noticed also that g(X) is closely related to the local entropy of the N data points. From the above definition it is easily observed that large g(**X**) values correspond to highly concentrated flat low noise regions. Another important situation is a well organized edge where both indexes g(**X**) and v(**X**) take large values an information exploited here very much.

The filtering process

Based on the above observations for the values of the two parameters we can roughly partition the image into the following 3 classes.

a) **The first class** is a well organized edge and v(**X**) as well as g(**X**) take large values.
b) **The second** one is a ramp where v(**X**) is large and g(**X**) is small.
c) **The third class** is a flat region where v(**X**) is small.

The fuzzy partition of the image to three classes is very informative for the optimum processing to be used.

In this work the objective is to enhance the image quality by filtering out Gaussian and Impulsive noise and maintain local features of the image. These objectives are met by adopting a **selective averaging** filtering process. Usually this selection is based on an ordering scheme. Dealing with vectors the ordering is accomplished by means of a ***reduced ordering*** technique. Usually the aggregate distance of each vector to the others is taken as distance measure. Regarding preservation of spatial information better results are received if the reference point (the center of the window) is used for the vector ordering. The price to be paid in this case is the extra operations needed to secure that the reference point is not an Impulse. This extra information can also be included in the algorithm, using soft or hard decision and transferring the control to the aggregate distance ordering or to the ordering according to the distance from the reference point. Preliminary results of the above algorithm are very promising . However in this paper we will describe only the simplified version of the algorithm where only ordering based on the aggregate distance is assumed.

The pixels of the window, i.e. the vectors $\mathbf{X}_i$s, (i=1, N) are ordered and a number of them is selected and averaged. If the most centroid vector is selected then the filter output is the well known **Vector Median Filter** (VMF). If all the vectors are selected then it is just a **moving Average Filter.** A selection of a number in-between these two extreme cases sometimes consists the best choice. The optimum selection depends on the specific class the image vectors $\mathbf{X}_i$ belong each time. To facilitate this process we define:

a) for the first class 1 pixel is selected as the filter output (VMF)
b) for the second class a small number of pixels are selected and averaged. Typically 1/3 of the total number is assumed.
c) for the third class all the pixels are selected and averaged.

Membership functions and fuzzy sets.

In this paper, two fuzzy sets (Large, Small) are proposed both for the dispersion v(**X**) and for the potential index g(**X**). These sets are shown in figure 1 and are defined by the values a,b,c.

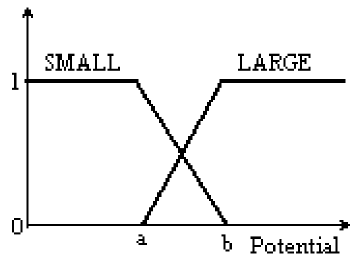

Fig. 1. Two fuzzy sets for p(X) and g(X) are considered: Large and Small. (The units are different)

Fuzzy rules link the conditions about the two parameters in the antecedents (i.e. p(**X**), g(**X**)), to conclusions about the consequent which are the imaging conditions and therefore the corresponding classes.

The formulation of the rules is the following:

Rule1:IF (v,Large) AND (g, Large) THEN (Class1)
Rule2:IF (v,Large) AND (g, Small) THEN (Class2)
Rule3:IF (v,Small) THEN (Class3)

Assuming $\mu_S(v)$, $\mu_L(v)$, $\mu_S(g)$, $\mu_L(g)$ membership values for the Small and Large sets of Dispersion and Potential index, the inference result for each rule is given by:

$$\mu_1 = \min(\mu_L(v), \mu_L(g))$$
$$\mu_2 = \min(\mu_L(v), \mu_S(g))$$
$$\mu_3 = \mu_S(v)$$

The complete inference result is constructed by the following defuziffication operation which is based on the weighted average method:

$$\mathrm{output}(X) = \frac{\sum_{m=1}^{3} \mu_m(X) \cdot y_m(X)}{\sum_{m=1}^{3} \mu_m(X)}$$

where $y_m(X)$ is the output of the suitable filter for class m.

The whole process is depicted in the block diagram of figure 2.

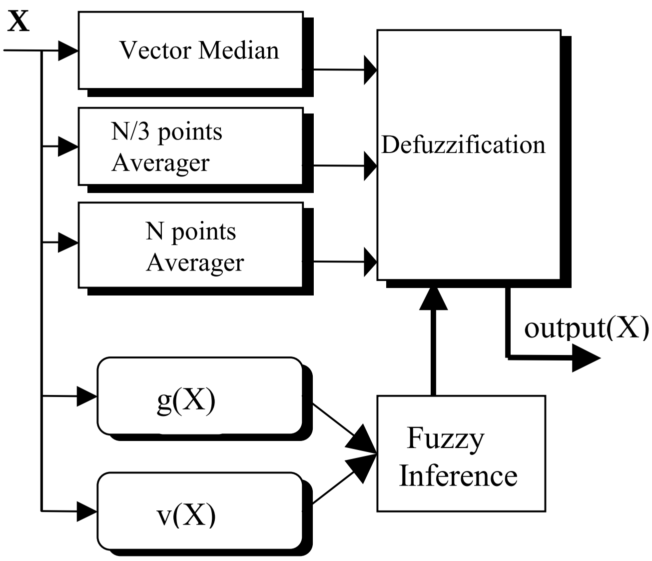

Fig2. The block diagram for the proposed filter.

Experimental results

The algorithm has been tuned by assigning values to parameters of the fuzzy sets experimentally.

The test image employed here is the PEPPERS. The noise added to the original image is mixed gaussian(0,655) and impulsive(1%). The proposed filter is also compared to the VMF and the average filter of the same window. Objective evaluation indicate superior performance.

A subjective evaluation is performed using the MSE computed as follows:

$$MSE = \frac{1}{NxM} \sum_{i=1}^{N} \sum_{i=1}^{M} (y(i,j) - x(i,j))^2$$

where y(i,j) are the pixels of the output, x(i,j) are the pixels of image without noise and N,M are the image dimension (in pixels).

Results of this comparison presented in TABLE I , indicate the optimized performance of the proposed fuzzy filters.

TABLE I

Filter type (3x3)	ó$_n$=18 impulse=0.8%	ó$_n$=26 impulse=1%
Averager	269	203
Vector Median	288	216
Proposed Filter	223	177

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. (a)

(b)

(c).

(d).

Figure 3.
a)Noisy image with gaussian (0,676) and impulsive (1 %)
b)The out put of VM
c)The output of the averager
d)The output of the proposed filter

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